

**CLAIMS****What Is Claimed:**

- 1 1. A method for simulating a multi-dimensional space, comprising:  
2 generating a sequence of pseudo-random numbers according to a  
3 prescribed quasi-Monte Carlo model; and  
4 mapping each pseudo-random number R of the sequence of  
5 random numbers into multiple variables of unique values for the multi-  
6 dimensional space, the multi-dimensional space including D dimensions,  
7 where D is a number.
- 1 2. The method of claim 1, further comprising assigning the unique values to  
2 each dimension based upon a prescribed index.
- 1 3. The method of claim 1, further comprising sampling the multiple variables  
2 of the multi-dimensional space and statistically analyzing the sampled  
3 multiple variables according to a prescribed error analysis.
- 1 4. The method of claim 1, further comprising sampling the multiple variables  
2 of the multi-dimensional space and performing numerical integrations  
3 upon the sampled multiple variables.
- 1 5. The method of claim 1, wherein each pseudo-random number R  
2 generated by the prescribed quasi-Monte Carlo model includes a floating  
3 point number having a value between 0.0 and 1.0, further wherein each  
4 dimension is characterized by a unique value based upon an index, the  
5 index equal to a total combinations of dimensional value points TC times a  
6 respective pseudo-random number R.

1 6. The method of claim 1, wherein each of the multiple variables of the multi-  
2 dimensional space represents a corresponding D dimension value and  
3 wherein each dimension is characterized by a minimum and a maximum  
4 value, further wherein each dimension is characterized by a prescribed  
5 resolution S.

1 7. The method of claim 6, wherein the D dimension values are further  
2 characterized by a first dimension D0 that includes minimum and  
3 maximum values defined as D0.min and D0.max, respectively, a second  
4 dimension D1 that includes minimum and maximum values defined as  
5 D1.min and D1.max, etceteras, up to a Dth dimension.

1 8. The method of claim 6, further comprising selecting a value of S according  
2 to a desired accuracy of a final simulation value, wherein the value of S  
3 defines a grid for use in conjunction with the mapping of the pseudo-  
4 random numbers into the multiple variables of the multi-dimensional  
5 space.

1 9. The method of claim 8, wherein selecting the value of S includes deriving  
2 the value of S such that a ratio  $r$ , as defined by  $r = s^D/P^N$ , is not factorable  
3 by one of the following selected from the group consisting of base P and  
4 the number of dimensions D, and where N is the number of pseudo-  
5 random numbers and  $r$  is a prescribed prime number.

10. A method for simulating a multi-dimensional space, comprising:
- generating a sequence of pseudo-random numbers according to a prescribed quasi-Monte Carlo model;
  - mapping each pseudo-random number  $R$  of the sequence of random numbers into multiple variables of unique values for the multi-dimensional space, the multi-dimensional space including  $D$  dimensions, where  $D$  is a number, wherein each of the multiple variables of the multi-dimensional space represents a corresponding  $D$  dimension value and wherein each dimension is characterized by a minimum and a maximum value, the  $D$  dimension values further being characterized by a first dimension  $D_0$  that includes minimum and maximum values defined as  $D_0.min$  and  $D_0.max$ , respectively, a second dimension  $D_1$  that includes minimum and maximum values defined as  $D_1.min$  and  $D_1.max$ , etceteras, up to a  $D_{th}$  dimension, further wherein each dimension is characterized by a prescribed resolution  $S$ ; and
  - selecting a value of  $S$  according to a desired accuracy of a final simulation value, wherein the value of  $S$  defines a grid for use in conjunction with the mapping of the pseudo-random numbers into the multiple variables of the multi-dimensional space, wherein selecting the value of  $S$  includes deriving the value of  $S$  such that a ratio  $r$ , as defined by  $r = s^D/P^N$ , is not factorable by one of the following selected from the group consisting of base  $P$  and the number of dimensions  $D$ , and where  $N$  is the number of pseudo-random numbers and  $r$  is a prescribed prime number.

- 1 11. A method for simulating trace impedance of a printed circuit board  
2 characterized by at least three dimensions of a multi-dimensional space,  
3 said method comprising:  
4 generating a sequence of pseudo-random numbers according to a  
5 prescribed quasi-Monte Carlo model; and  
6 mapping each pseudo-random number R of the sequence of  
7 random numbers into multiple variables of unique values for the multi-  
8 dimensional space, the multi-dimensional space including D dimensions,  
9 where D is a number.
- 1 12. The method of claim 11, further comprising assigning the unique values to  
2 each dimension based upon a prescribed index.
- 1 13. The method of claim 11, further comprising sampling the multiple variables  
2 of the multi-dimensional space and statistically analyzing the sampled  
3 multiple variables according to a prescribed error analysis.
- 1 14. The method of claim 11, further comprising sampling the multiple variables  
2 of the multi-dimensional space and performing numerical integrations  
3 upon the sampled multiple variables.
- 1 15. The method of claim 11, wherein each pseudo-random number R  
2 generated by the prescribed quasi-Monte Carlo model includes a floating  
3 point number having a value between 0.0 and 1.0, further wherein each  
4 dimension is characterized by a unique value based upon an index, the  
5 index equal to a total combinations of dimensional value points TC times a  
6 respective pseudo-random number R.

- 1 16. The method of claim 11, wherein each of the multiple variables of the  
2 multi-dimensional space represents a corresponding D dimension value  
3 and wherein each dimension is characterized by a minimum and a  
4 maximum value, further wherein each dimension is characterized by a  
5 prescribed resolution S.
- 1 17. The method of claim 16, wherein the D dimension values are further  
2 characterized by a first dimension D0 that includes minimum and  
3 maximum values defined as D0.min and D0.max, respectively, a second  
4 dimension D1 that includes minimum and maximum values defined as  
5 D1.min and D1.max, etceteras, up to a Dth dimension.
- 1 18. The method of claim 16, further comprising selecting a value of S  
2 according to a desired accuracy of a final simulation value, wherein the  
3 value of S defines a grid for use in conjunction with the mapping of the  
4 pseudo-random numbers into the multiple variables of the multi-  
5 dimensional space.
- 1 19. The method of claim 18, wherein selecting the value of S includes deriving  
2 the value of S such that a ratio  $r$ , as defined by  $r = s^D/P^N$ , is not factorable  
3 by one of the following selected from the group consisting of base P and  
4 the number of dimensions D, and where N is the number of pseudo-  
5 random numbers and  $r$  is a prescribed prime number.

1 20. Apparatus for simulating trace impedance of a printed circuit board, the  
2 printed circuit board characterized by at least three dimensions of a multi-  
3 dimensional space, said apparatus comprising:

4 a random number generator for generating a sequence of pseudo-  
5 random numbers according to a prescribed quasi-Monte Carlo model;

6 a mapping processor for mapping each pseudo-random number R  
7 of the sequence of random numbers into multiple variables of unique  
8 values for the multi-dimensional space, the multi-dimensional space  
9 including D dimensions, where D is a number, wherein each of the  
10 multiple variables of the multi-dimensional space represents a  
11 corresponding D dimension value and wherein each dimension is  
12 characterized by a minimum and a maximum value, the D dimension  
13 values further being characterized by a first dimension D0 that includes  
14 minimum and maximum values defined as D0.min and D0.max,  
15 respectively, a second dimension D1 that includes minimum and  
16 maximum values defined as D1.min and D1.max, etceteras, up to a Dth  
17 dimension, further wherein each dimension is characterized by a  
18 prescribed resolution S; and

19 a value selector for selecting a value of S according to a desired  
20 accuracy of a final simulation value, wherein the value of S defines a grid  
21 for use in conjunction with the mapping of the pseudo-random numbers  
22 into the multiple variables of the multi-dimensional space, wherein  
23 selecting the value of S includes deriving the value of S such that a ratio r,  
24 as defined by  $r = s^D/P^N$ , is not factorable by one of the following selected  
25 from the group consisting of base P and the number of dimensions D, and  
26 where N is the number of pseudo-random numbers and r is a prescribed  
27 prime number.

21. A method of manufacturing a printed circuit board comprising:

characterizing the printed circuit board by at least three dimensions of a multi-dimensional space; and

manufacturing the printed circuit board in accordance with a simulated trace impedance, the simulated trace impedance obtained by:

generating a sequence of pseudo-random numbers according to a prescribed quasi-Monte Carlo model;

mapping each pseudo-random number R of the sequence of random numbers into multiple variables of unique values for the multi-dimensional space, the multi-dimensional space including D dimensions, where D is a number, wherein each of the multiple variables of the multi-dimensional space represents a corresponding D dimension value and wherein each dimension is characterized by a minimum and a maximum value, the D dimension values further being characterized by a first dimension D0 that includes minimum and maximum values defined as D0.min and D0.max, respectively, a second dimension D1 that includes minimum and maximum values defined as D1.min and D1.max, etceteras, up to a Dth dimension, further wherein each dimension is characterized by a prescribed resolution S; and

selecting a value of S according to a desired accuracy of a final simulation value, wherein the value of S defines a grid for use in conjunction with the mapping of the pseudo-random numbers into the multiple variables of the multi-dimensional space, wherein selecting the value of S includes deriving the value of S such that a ratio r, as defined by  $r = s^D / P^N$ , is not factorable by one of the following selected from the group consisting of base P and the number of dimensions D, and where N is the number of pseudo-random numbers and r is a prescribed prime number.

1 22. A computer system, comprising:

2 a printed circuit board manufactured in accordance with a simulated  
3 trace impedance, said printed circuit board including impedance traces  
4 that characterize at least three dimensions of a multi-dimensional space of  
5 said printed circuit board, wherein said impedance traces include trace  
6 impedances obtained by:

7 generating a sequence of pseudo-random numbers  
8 according to a prescribed quasi-Monte Carlo model;

9 mapping each pseudo-random number R of the sequence of  
10 random numbers into multiple variables of unique values for the  
11 multi-dimensional space, the multi-dimensional space including D  
12 dimensions, where D is a number, wherein each of the multiple  
13 variables of the multi-dimensional space represents a  
14 corresponding D dimension value and wherein each dimension is  
15 characterized by a minimum and a maximum value, the D  
16 dimension values further being characterized by a first dimension  
17 D0 that includes minimum and maximum values defined as D0.min  
18 and D0.max, respectively, a second dimension D1 that includes  
19 minimum and maximum values defined as D1.min and D1.max,  
20 etceteras, up to a Dth dimension, further wherein each dimension is  
21 characterized by a prescribed resolution S; and

22 selecting a value of S according to a desired accuracy of a  
23 final simulation value, wherein the value of S defines a grid for use  
24 in conjunction with the mapping of the pseudo-random numbers  
25 into the multiple variables of the multi-dimensional space, wherein  
26 selecting the value of S includes deriving the value of S such that a  
27 ratio r, as defined by  $r = s^D/P^N$ , is not factorable by one of the  
28 following selected from the group consisting of base P and the  
29 number of dimensions D, and where N is the number of pseudo-  
30 random numbers and r is a prescribed prime number.